V. *Data for the Problem of Evolution in Man.—VI. A First Study of the Correlation of the Human Skull.*

By Alice Lee, *D.Sc., with some Assistance from* Karl Pearson, *F.R.S., University College, London.*

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NOTE.

The substance of this paper was presented by Miss Lee as a thesis for the London D.Sc. in March, 1899. After its presentation Miss Lee asked me to criticise and revise it with a view to publication. Illness in the spring of 1899 and later pressure of other work prevented my completing this revision until now. When Miss Lee started her work practically nothing had been published on the correlation of the parts of the skull; since then an interesting paper has appeared by Dr. Franz Boas. To this reference is made in the footnotes at points where there is agreement or disagreement with his conclusions. The subject is of such great scientific interest, and anthropologically of such importance, that I urged Miss Lee to somewhat enlarge her original thesis by a series of additional investigations now incorporated in this paper. I have further rearranged a good deal of her material and reworded some of her conclusions, but the reduction of the material and the inferences drawn from it are substantially her work. My task has been that of an editor, who wished to mould the author's researches into a component part of a wider series dealing generally with the quantitative data for the problem of evolution in man. Such is the limit of my revision. I have passed of course nothing which did not seem to me valid, and have suggested to the author some lacunæ which could be filled up by a consideration of additional data.—Karl Pearson.

Contents.

(1.) THE reconstruction of an organism from a knowledge of some only of its parts is a problem which has occupied the attention of biologists for many years past. CUVIER was the first to introduce in his 'Discours sur les Révolutions de la Surface du Globe,' 1812,* the idea of correlation. He considered that a knowledge of the size of a shoulder blade, leg, or arm might make it possible to reconstruct the whole individual to which the bone had belonged. The conception was taken up by OWEN, but has fallen into discredit owing to the many errors made in attempts from a wide but only *qualitative* knowledge of the skeleton, to reconstruct forms the appreciation of which depends really on *quantitative* measurement and an elaborate quantitative theory. Such a theory having now been developed, and anatomists having provided large series of measurements, it has become possible to reconsider the problem on a sounder basis, and to determine more closely the limits under which our modern methods may be safely applied.

The three fundamental problems of the subject are: (i.) The reconstruction of an individual, of whom one or more organs only are known, when a series of organs for individuals of the same local race have been measured and correlated.

As illustration, one may take the reconstruction of the probable stature of an individual for medico-legal purposes when a limb only has been found.

(ii.) The reconstruction of the mean type of a local race from a knowledge of a series of one or more organs in that race, when a wide series of these and other organs have been measured in other races.

As illustration, we may consider the reconstruction of the stature of prehistoric and defunct races from the measurement of their long bones, when the correlations between stature and long bones for some modern race have been determined from measurements made in the dissecting room.†

An important question in all researches of this kind is the legitimacy of applying results obtained for one local race to a second. We know that the variability and

* Page 98 of the edition of 1830, the earliest in our Library.

† See PEARSON, "On the Stature of Prehistoric Races," 'Phil. Trans.,' A, vol. 192, pp. 169–244. An attempt is now being made by Professors WINDLE and PEARSON to collect data from English dissecting rooms, and an elaborate series of measurements with the like end in view are now being made in Strasburg on German material.

correlation are not constant for all the local races of a species; some of the limits of this legitimacy will be considered in this paper. A very full discussion of the matter for the regression equations of the long bones in the case of twenty local races in man by Mr. LESLIE BRAMLEY-MOORE is nearly completed.

(iii.) The reconstruction of an organ in the living individual not measurable during life, from a determination of the size of accessible organs, and a knowledge of the correlation between these organs and the inaccessible organ obtained from measurements made on individuals of the same race after death.

As an illustration, we may take the determination of the skull capacity from measurements made on the head of living individuals.

In all the three problems cited above, we can only obtain *probable* results, *i.e.*, we obtain the average value—generally not very far from the modal value of the second organ in a group of individuals with their first organ equal to that of the particular individual measured. The closeness of the result obtained is determined fairly accurately by the probable error of the array or group of individuals above referred to. If, instead of reconstructing an individual, we reconstruct a local race from a fairly large number of organs, this probable error will be at once largely reduced; but in doing this we assume the legitimacy of applying results obtained from one local race to a second local race.

(2.) The whole theory of reconstruction is summed up in the determination of the regression equations. It has been shown* that the most probable value of an organ, B, reconstructed from n organs $A_1, A_2 \ldots . A_n$, is given by the expression

$$B - m_0 = -\left\{\frac{R_{01}}{R_{00}}\frac{\sigma_0}{\sigma_1}(A_1 - m_1) + \frac{R_{20}}{R_{00}}\frac{\sigma_0}{\sigma_2}(A_2 - m_2) + \ldots + \frac{R_{0n}}{R_{00}}\frac{\sigma_0}{\sigma_n}(A_n - m_n)\right\} \quad \text{(i.)},$$

with a probable error $= {\cdot}67449\sigma_0\sqrt{R/R_{00}}$ . . . . (ii.),

where r_{0q} = correlation coefficient of B and A_q,

$r_{q'q} = r_{qq'}$ = ,, ,, ,, A_q ,, $A_{q'}$,

σ_0 = standard deviation of B,

σ_q = ,, ,, ,, A_q,

m_0 = mean of B,

m_q = ,, A_q,

$-\frac{R_{0q}}{R_{00}}\frac{\sigma_0}{\sigma_q}$ = partial regression coefficient of B from A_q,

and R is the following determinant, R_{pq} the minor corresponding to r_{pq}:

$$\begin{vmatrix} 1, & r_{01}, & r_{02}, & \cdots & r_{0n} \\ r_{10}, & 1, & r_{12}, & \cdots & r_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{n0}, & r_{n1}, & r_{n2} & \cdots & 1 \end{vmatrix}$$

* 'Phil. Trans.,' A, vol. 192, p. 172.

All the regression equations in the present paper have been worked from the above results, the most lengthy being those which depended on the evaluation of the numerical magnitude of the above determinant for the correlation of four organs.

(3.) The special object of this investigation is to apply the theory given in the last section to the reconstruction of skull capacity—to determine which of the measurements, length, breadth, height, or cephalic index of skull, or which combination of these measurements, will give the best result. In carrying out this special investigation, all the three fundamental problems considered in section (1) naturally arise, and will be referred to below. Further, certain problems regarding variation and correlation in man and woman also occur, and will be considered in their places.

The problem of the determination of the capacity of the skull has been one which has long occupied the attention of craniologists and anthropologists, and a great variety of methods have been considered and have found acceptance from one or another authority. The ideal method has not, however, been yet discovered, and in the well-known 'Frankfurter Verständigung' of the German craniologists, the matter was reserved for "further consideration," and has remained for a number of years in that unsettled state. From a fairly elaborate system of skull capacity measurements made at University College by Miss C. D. FAWCETT, B.Sc., it would appear that the same experimenter may, with very slight practice, reach surprisingly close results for the capacity by very diverse methods; but that two different experimenters may give a mean skull capacity for a series which differs by 15 to 40 cub. centims. This of course only denotes about 1 to 3 per cent. of personal equation; but it appears large when read in gross. I cannot think that any conclusions as to relative racial differences ought to be based solely on divergencies in skull capacity of less than 40 cub. centims. when the two or more series under consideration have been measured by different observers. The knowledge of this divergence arising from the personal equation of different observers has led certain craniologists to suggest formulæ for calculating the capacity of the skull without measuring its contents, but from measurements of its girth, its height, length, or breadth. These formulæ seem to be unsatisfactory because they have not been based on a knowledge of the mathematical theory of correlation. It will be shown in the sequel that a formula can be found which gives the average capacity of a series of skulls from their mean height, length, and breadth with a fair degree of accuracy. In view of this it is a matter for consideration whether its use might not effectively replace the laborious and unsatisfactory methods of determining capacity by seed, shot, or sand. These could always be fallen back upon should any suspicion arise that the formula in question was being applied to a too widely divergent local race.

(4.) In selecting material for this investigation, I had to bear in mind results already reached by my co-workers at University College, but only in part at present published. In particular, that there was comparatively small correlation between the parts of the skull usually measured, and, further, that such correlation as actually exists varies enormously, even in sign, as we pass from one local race to another. This want of correlation, or want of steady correlation, in the parts of the skull, as compared with the correlation exhibited by the long bones, or by parts of the hand, is extremely interesting from the standpoint of evolution. It would appear to be much easier to modify a single character of the skull by selection without altering other characters than can be the case with other parts of the skeleton.

The measurements considered in this paper are: L the greatest length, B the greatest breadth of the skull, H the height measured from the auricular line, I the cephalic index = B/L, and C the capacity. In choosing the material several points had to be borne in mind:

(i.) A sufficiently large series must be used.

As a matter of fact, 50 to 100 skulls are considered by craniologists to be a fair series, but such numbers are small from the mathematician's standpoint.

(ii.) Material must be drawn from as widely different races as possible, if we are to measure the legitimacy of applying results obtained from one local race to another.

(iii.) The capacities must have been determined by competent observers using approximately like methods of measurement.

The data which seemed to me to approximately fulfil these conditions are the following:—

(*a*.) A series of Bavarian (*Alt-Baierische*) skulls measured by Professor J. RANKE, and given in his 'Beiträge zur physischen Anthropologie der Bayern.' In this case there were 100 ♂ and 99 ♀ with L, B, H, I, and C available.

(*b*.) A series of Aino skulls measured by KOGANEI, a craniologist trained in German schools, and given in the 'Mittheilungen aus der Medicinischen Facultät der Kaiserlich-Japanischen Universität,' Tokio, Bd. ii., 1894. In this case L, B, H, I, and C are given for 76 ♂ and 52 ♀ skulls, and there are 11 ♂ and 11 ♀ skulls for which L, B, H, and I only are given.

(*c*.) A series of Naqada skulls discovered in Egypt by Professor FLINDERS PETRIE, and measured by Miss C. D. FAWCETT, B.Sc., on the basis of the 'Frankfurter Verständigung.' I have to thank her for allowing me to use her results before publication. In this case L, B, H, and C were available for 69 ♂ and 98 ♀ skulls, and L, B, H only for 76 ♂ and 100 ♀ skulls.

As supplementary and test series, I have used primarily—

(*d*.) 201 ♂ and 96 ♀ skulls of ancient Egyptians. This series consists of mummies from Thebes in the Mook collection at Leipzig.

(*e*.) 76 ♂ and 23 ♀ skulls of modern Egyptians in a *Privat-Sammlung* at Leipzig.

The details of both these series are taken from the great craniological catalogue of the German Anthropological Society.*

(5.) Starting with the series (*a*) and (*b*), I have obtained for their means and

* The parts are published separately as off-prints from the 'Archiv für Anthropologie.'

variabilities the results in Table I. below. In this case the Aino may be looked upon as a primitive uncivilised and the Germans as a highly-developed civilised race. An examination of this table shows that the Germans while gaining in breadth have lost in length, the mean auricular height for both sexes in both races remaining fairly stationary. Thus the brachycephalic tendency is in this case accompanied by a loss of length, and is not merely a gain in breadth.

TABLE I.

		Mean.	Standard deviation.	Coefficient of variation.
Aino, male . .	Length	185·82 millims.	5·936	3·195
	Breadth	141·23	3·897	2·759
	Height	119·32	4·377	3·668
	Capacity. . . .	1461·64 cub. centims.	100·605	6·883
	Cephalic index .	76·50	2·392	3·127
Aino, female .	Length	177·17 millims.	5·453	3·077
	Breadth	136·79	3·662	2·677
	Height	114·97	3·651	3·175
	Capacity . . .	1307·69 cub. centims.	89·751	6·864
	Cephalic index .	77·40	2·440	3·152
German, male .	Length	180·58 millims.	6·088	3·371
	Breadth	150·47	5·849	3·887
	Height	120·75	5·397	4·469
	Capacity . . .	1503·72 cub. centims.	116·890	7·773
	Cephalic index .	83·30	3·500	4·201
German, female	Length	173·59 millims.	6·199	3·571
	Breadth	144·11	4·891	3·394
	Height	114·17	4·463	3·909
	Capacity . . .	1337·15 cub. centims.	108·730	8·131
	Cephalic index	83·10	2·973	3·578

It will further be seen that the Aino are less variable than the Germans in *all* the characters under discussion,* and in both sexes. The increase in skull capacity of the Germans on the Aino is less for the female than for the male, whilst in the variation of this character the change is greater for the female than the male. Further, the variability of the two sexes is more nearly equal in the Ainos than in the Germans.

These results are in good accord with those obtained by KARL PEARSON in his paper on "Variation in Man and Woman," and by him and myself in our paper "On the Relative Variation and Correlation in Civilised and Uncivilised Races," the conclusions there reached being—

(*a*.) Civilised races are more variable than uncivilised races.

* It must be noted that the Germans are not a town population, but skulls from the churchyard mortuary chapels (*Gebein-Häuser*) of a limited rural district.

(*b.*) There is greater equality of variation for the two sexes in uncivilised than in civilised races.

(*c.*) Man tends with advance in civilisation to gain in size on woman.

(*d.*) Woman tends with advance in civilisation to gain in variability on man.

(6.) Turning to the correlations we obtain for Aino and Germans the results given in Table II. The correlation-coefficients are clearly very different for the two races. Putting aside the somewhat erratic correlation of capacity and cephalic index, we note that for the Aino the female correlations are all less than the male, but for the

TABLE II.—Coefficients of Correlation.

Organs.	Male Aino.		Female Aino.	
Capacity and length $= r_{01}$	·8928 ± ·0157	No. = 76	·6627 ± ·0525	No. = 52
Capacity and breadth $= r_{02}$	·5606 ± ·0531		·5021 ± ·0700	
Capacity and height $= r_{03}$	·5444 ± ·0544		·5210 ± ·0681	
Capacity and cephalic index	− ·3069 ± ·0701		− ·2466 ± ·0878	
Length and breadth $= r_{12}$	·4316 ± ·0588	No. = 87	·3765 ± ·0729	No. = 63
Length and height $= r_{13}$	·5008 ± ·0542		·3489 ± ·0746	
Breadth and height $= r_{23}$	·3454 ± ·0637		·1778 ± ·0823	
	Male German.		**Female German.**	
Capacity and length $= r_{01}$	·5152 ± ·0495	No. = 100	·6873 ± ·0360	No. = 69
Capacity and breadth $= r_{02}$	·6720 ± ·0370		·7068 ± ·0339	
Capacity and height $= r_{03}$	·2431 ± ·0635		·4512 ± ·0540	
Capacity and cephalic index	·2022 ± ·0647		− ·0307 ± ·0677	
Length and breadth $= r_{12}$	·2861 ± ·0619		·4876 + ·0517	
Length and height $= r_{13}$	− ·0975 ± ·0668		·3136 ± ·0611	
Breadth and height $= r_{23}$	·0715 ± ·0671		·2764 ± ·0626	

German the female are all greater than the male. Further, with the same omission in five out of the six cases, the Aino male is more highly correlated than the German male, and in four out of the six cases the German female is more highly correlated than the Aino female. This is again in general agreement with the results suggested in the second paper cited above, namely :—*

(*a.*) That correlation is more nearly equal for the two sexes in uncivilised than in civilised races, and

(*b.*) That woman tends with advance to gain in correlation on man.

* This confirmation of the results of the above paper is of interest, since they have been called in question by E. T. BREWSTER ('Proc. Boston Soc., Nat. Hist.' vol. 29, pp. 45–61). His series, however, are extremely small and his treatment of them not entirely satisfactory.

In the Aino race the length is more highly correlated with the capacity than the other dimensions are. In the German race, on the other hand, it is the breadth. Thus we shall find for the Ainos that formulæ involving the length, and for the Germans that formulæ involving the breadth, give the least probable error in the reconstruction of the capacity. It would be of interest to investigate whether this result is a distinguishing mark of dolichocephalic and brachycephalic races.

The correlation of capacity and cephalic index is, as I have said, somewhat erratic. For the Aino male and female it is quite sensible but negative. In other words, in a dolichocephalic race, it would appear as if dolichocephaly tended towards greater skull capacity. On the other hand, among the brachycephalic Germans, there is for the males a sensible correlation of a positive kind between capacity and brachycephaly. For the German women, however, we find this correlation less than half the probable error, and thus practically non-existent.

In order to throw, if possible, more light on this point the results in Table III. were worked out for two races, one of which is rather more dolichocephalic than the Aino. In this case very little stress can be laid on the ♀ modern Egyptians; they are far too few in number. The ♀ Theban mummies give a sensibly zero correlation, but in the three other cases the correlation is clearly negative. Thus there appears to be little doubt that in dolichocephalic races those who possess the race character most markedly have the greater skull capacity.

TABLE III.

	Mean.	Standard deviation.	Correlation.	Number.
Male Thebans (Mummies):				
Capacity	1393·6	120·80	− ·1480 ± ·0482	187
Cephalic index	74·8	3·17		
Female Thebans (Mummies):				
Capacity	1248·2	102·02	+ ·0080 ± ·0736	84
Cephalic index	76·3	3·70		
Male Modern Egyptians:				
Capacity	1356·5	116·55	− ·1410 ± ·0883	56
Cephalic index	77·3	5·42		
Female Modern Egyptians:				
Capacity	1195·8	85·74	− ·4960 ± ·1060	23
Cephalic index	76·7	5·10		

In Table IV. will be found similar data for three fairly brachycephalic races:—

TABLE IV.

Race.	Mean.	Standard deviation.	Correlation.	Number.
Male French:				
Capacity	1473·05	107·33	·1437 ± ·0883	56
Cephalic index	79·8	4·078		
Male Malays:				
Capacity	1429·76	100·243	·0331 ± ·0773	76
Cephalic index	81·9	5·127		
Male Etruscans:				
Capacity	1455·9	135·87	·2157 ± ·0729	78
Cephalic index	78·5	3·322		
Female Etruscans:				
Capacity	1323·6	110·77	·1443 ± ·1071	38
Cephalic index	78·3	3·300		

We see that the correlation is in all cases positive, but it is small, and in three of the cases given is hardly sensible considering the size of the probable errors. On the whole, I think we must conclude that while there is only a small relationship between cephalic index and capacity, yet that in brachycephalic races greater roundness points to greater capacity, and in dolichocephalic races less roundness points to greater capacity. In either case the emphasis of the racial character denotes an increase of capacity.

Accordingly, while we have been able to draw some interesting general conclusions as to the relationship of brachycephaly and capacity, it will be clear that the correlation here is far too uncertain to base any reliable reconstruction formula upon it. The regression formula for capacity in this case will be found to have, on the whole, the largest probable error, and to give the worst results when applied to test cases selected at random.*

(7.) I turn to the general regression formulæ for the determination of capacity. These are given for the Aino and Germans of both sexes in Tables V. to VIII. It will be seen from these tables that the reconstruction formulæ based on the cephalic index has in each case the largest probable error. Further, a very slight examination of the tables confirms the remark already made that for the Aino the length is a more important factor than the breadth, and that for the Germans the breadth is more important than the length as far as capacity is concerned. In the former race,

* The general result as to cephalic index agrees with that obtained by Dr. FRANZ BOAS, 'American Anthropologist,' N.S., July, 1899, "The Cephalic Index," p. 448.

formulæ involving the length give, on the whole, a lower probable error in the value calculated for the capacity; in the latter race we must replace in this statement length by breadth.*

TABLE V.

	Formulæ for Aino males.	Probable error of mean.
(1)	$C = 15{\cdot}130\,L - 1349{\cdot}95$	$\frac{30{\cdot}57}{\sqrt{n}}$
(2)	$C = 14{\cdot}472\,B - 582{\cdot}24$	$\frac{56{\cdot}19}{\sqrt{n}}$
(3)	$C = 12{\cdot}511\,H - 31{\cdot}21$	$\frac{56{\cdot}92}{\sqrt{n}}$
(4)	$C = -12{\cdot}907\,I + 2449{\cdot}00$	$\frac{64{\cdot}58}{\sqrt{n}}$
(5)	$C = 13{\cdot}555\,L + 5{\cdot}562\,B - 1842{\cdot}61$	$\frac{27{\cdot}58}{\sqrt{n}}$
(6)	$C = 14{\cdot}029\,L + 2{\cdot}984\,H - 1501{\cdot}23$	$\frac{29{\cdot}61}{\sqrt{n}}$
(7)	$C = 10{\cdot}921\,B + 9{\cdot}153\,H - 1172{\cdot}95$	$\frac{50{\cdot}14}{\sqrt{n}}$
(8)	$C = 12{\cdot}857\,L + 5{\cdot}171\,B + 2{\cdot}190\,H - 1919{\cdot}24$	$\frac{27{\cdot}02}{\sqrt{n}}$
(9)	$C = {\cdot}000328\,(L \times B \times H) + 430{\cdot}30$	$\frac{42{\cdot}89}{\sqrt{n}}$

Capacity is measured in cubic centims., and length, breadth, and height in millims.
n = number from which C is determined.
I = cephalic index = $100\,B/L$

* Dr. FRANZ BOAS (*loc. cit.*, p. 461) states: "The relation between capacity and head-diameters is found to be of fundamental importance, and among these the relation between transversal diameter and capacity is most significant." This, I think, is only true for fairly brachycephalic races. He is dealing with 87 Sioux Indian skulls with cephalic index = 79.

TABLE VI.

Formulæ for Aino females.	Probable error of mean.
(1) $C = 10\cdot908\ L - 624\cdot86$	$\frac{45\cdot33}{\sqrt{n}}$
(2) $C = 12\cdot334\ B - 375\cdot63$	$\frac{52\cdot35}{\sqrt{n}}$
(3) $C = 12\cdot809\ H - 164\cdot95$	$\frac{51\cdot67}{\sqrt{n}}$
(4) $C = -9\cdot071\ I + 2028\cdot00$	$\frac{58\cdot67}{\sqrt{n}}$
(5) $C = 9\cdot084\ L + 7\cdot210\ B - 1288\cdot10$	$\frac{42\cdot22}{\sqrt{n}}$
(6) $C = 9\cdot013\ L + 8\cdot112\ H - 1221\cdot74$	$\frac{41\cdot29}{\sqrt{n}}$
(7) $C = 10\cdot363\ B + 10\cdot961\ H - 1370\cdot10$	$\frac{45\cdot12}{\sqrt{n}}$
(8) $C = 7\cdot379\ L + 6\cdot795\ B + 7\cdot752\ H - 1820\cdot40$	$\frac{46\cdot42}{\sqrt{n}}$
(9) $C = \cdot000400\ (L \times B \times H) + 187\cdot80$	$\frac{37\cdot90}{\sqrt{n}}$

TABLE VII.

Formulæ for German males.	Probable error of mean.
(1) $C = 9\cdot892\ L - 282\cdot55$	$\frac{67\cdot58}{\sqrt{n}}$
(2) $C = 13\cdot432\ B - 517\cdot34$	$\frac{58\cdot39}{\sqrt{n}}$
(3) $C = 5\cdot264\ H + 868\cdot05$	$\frac{76\cdot48}{\sqrt{n}}$
(4) $C = 6\cdot754\ I + 941\cdot11$	$\frac{77\cdot22}{\sqrt{n}}$
(5) $C = 6\cdot752\ L + 11\cdot421\ B - 1434\cdot06$	$\frac{51\cdot99}{\sqrt{n}}$
(6) $C = 10\cdot446\ L + 6\cdot414\ H - 1157\cdot17$	$\frac{63\cdot46}{\sqrt{n}}$
(7) $C = 13\cdot152\ B + 4\cdot245\ H - 987\cdot76$	$\frac{55\cdot47}{\sqrt{n}}$
(8) $C = 7\cdot348\ L + 10\cdot898\ B + 5\cdot228\ H - 2094\cdot31$	$\frac{75\cdot97}{\sqrt{n}}$
(9) $C = \cdot000332\ (L \times B \times H) + 415\cdot34$	$\frac{55\cdot41}{\sqrt{n}}$

TABLE VIII.

	Formulæ for German females.	Probable error of mean.
(1)	$C = 12{\cdot}055\ L - 755{\cdot}53$	$\frac{53{\cdot}27}{\sqrt{n}}$
(2)	$C = 15{\cdot}716\ B - 927{\cdot}66$	$\frac{51{\cdot}88}{\sqrt{n}}$
(3)	$C = 10{\cdot}993\ H + 82{\cdot}13$	$\frac{65{\cdot}45}{\sqrt{n}}$
(4)	$C = -1{\cdot}125\ I + 1430{\cdot}60$	$\frac{73{\cdot}31}{\sqrt{n}}$
(5)	$C = 7{\cdot}884\ L + 10{\cdot}842\ B - 1593{\cdot}96$	$\frac{43{\cdot}16}{\sqrt{n}}$
(6)	$C = 10{\cdot}618\ L + 6{\cdot}366\ H - 1232{\cdot}85$	$\frac{58{\cdot}70}{\sqrt{n}}$
(7)	$C = 14{\cdot}014\ B + 6{\cdot}749\ H - 1452{\cdot}89$	$\frac{48{\cdot}06}{\sqrt{n}}$
(8)	$C = 7{\cdot}065\ L + 10{\cdot}126\ B + 4{\cdot}848\ H - 1902{\cdot}02$	$\frac{40{\cdot}76}{\sqrt{n}}$
(9)	$C = {\cdot}000383\ (L \times B \times H) + 242{\cdot}19$	$\frac{42{\cdot}58}{\sqrt{n}}$

It will be noticed that a formula, No. (9), not hitherto referred to, has been introduced into these tables. As capacity is of three dimensions, an attempt has been several times made by anatomists to determine a relation between capacity and the product, L × B × H. This attempt seems to me to have failed because it has been based on a relation of the kind

$$\text{capacity} = \text{constant} \times (L \times B \times H),^*$$

whereas the mathematical theory shows that we should rather expect a relation of the type

$$\text{capacity} = \text{constant} + \text{constant} \times (L \times B \times H),$$

Of course, if L, B, and H differ only by small quantities, x_1, x_2, x_3, from their means, the last relation may be written

$$\text{capacity} = \gamma_0 + \gamma_1(m_1 + x_1)(m_2 + x_2)(m_3 + x_3),$$

where γ_0 and γ_1 represent constants, or

$$\begin{aligned} C &= \gamma_0 + \gamma_2 x_1 + \gamma_3 x_2 + \gamma_4 x_3 + \text{products of small quantities} \\ &= \gamma_5 + \gamma_2\ L + \gamma_3\ B + \gamma_4 H, \end{aligned}$$

where γ_5, γ_2, γ_3, γ_4 are constants, if we neglect terms of the order $x_1/m_1 \times x_2/m_2$ as compared with x_1/m_1 and x_2/m_2, &c.

* Relations of the form: capacity = const. × $(L + B + H)^3$ have also been suggested.

This would simply throw us back on the regression formula (8) of our tables. Now the order of x/m is of the order of σ/m, or $\frac{1}{100}$ the coefficient of variation, say, ·03 to ·04. But deviations equal to 3σ may and do occur; hence, in individual cases an error of 9 to 12 per cent. might arise, if we were to assume that formula (9) can be replaced in all cases by (8). Accordingly, at Mr. YULE's suggestion, I formed the product of L × B × H, and correlated this product with the capacity. In the following Table IX. I give the data for Aino, German, and Naqada races :—

TABLE IX.

	Mean L × B × H.	Standard deviation.	Coefficient of variation.	Coefficient of correlation L × B × H and capacity.
Male, Aino	3144286·72	237683·63	7·559	·7949 ± ·0389
Female, Aino . . .	2797031·90	174791·20	6·249	·7797 ± ·0367
Male, German . . .	3282337·66	246992·49	7·525	·7006 ± ·0343
Female, German . .	2860212·85	231245·01	8·084	·8142 ± ·0229
Male, Naqada . . .	2881136·61	199446·14	6·922	·6736 ± ·0443
Female, Naqada . .	2619630·70	179387·60	6·864	·7934 ± ·0253

I supplement this table by the remaining data required for the three races from Egypt :—

TABLE X.

Race.	No.	Capacity.		L × B × H.	
		Mean.	Standard deviation.	Mean.	Standard deviation.
Ancient Egyptians, .	201	1389·6	120·80*	2859374·1	†
Ancient Egyptians, ♀ .	96	1253·7	102·02*	2589814·6	†
Modern Egyptians, ♂ .	76	1354·5	116·55*	2801989·8	†
Modern Egyptians, ♀ .	23	1195·8	85·74*	2424920·4	†
Naqada, ♂	69	1386·6	104·36	2881136·6	199446·1
Naqada, ♀	98	1279·3	94·03	2619630·7	179387·6

For the Naqada race we deduce the regression formulæ from the above results :—

Males . . . . $C = {\cdot}000352 \times (L \times B \times H) + 372{\cdot}39.$

Females . . . $C = {\cdot}000416 \times (L \times B \times H) + 189{\cdot}81.$

The probable errors for reconstruction by aid of these are :—

Males . . $\dfrac{52{\cdot}0228}{\sqrt{n}}$. Females . . $\dfrac{38{\cdot}6026}{\sqrt{n}}$.

* Values cited from the smaller series in Table III. † Not calculated.

We have now all the data necessary for reconstructing the skull capacity, and it remains for us to consider how we can apply these to our three fundamental problems.

(8.) *First Fundamental Problem. The Reconstruction of the Individual from the known Formulæ for his own Race.*

In order to illustrate the degree of exactness with which we can reconstruct the individual from their own racial data, a perfectly random selection of twenty skulls was taken out of those of each sex for the Ainos and Germans, and their capacities reconstructed from each of the nine regression formulæ given in Tables V. to VIII. respectively. The results are tabulated in the following Tables XI. to XIV., and will enable the reader to appreciate the degree of exactness with which it would be possible to reconstruct the capacity of an individual skull from any one or more measurements made upon it.

These results show us at once that the last five formulæ are, when available, by far the best to use. (3) and (4), namely, reconstruction from the auricular height and the cephalic index, give occasionally very poor results. The latter formula, while of much interest from the racial standpoint,* need never be used for reconstruction, for the knowledge of the cephalic index means a knowledge of L and B, and accordingly we can always use (5) if not (8).

An examination of the actual mean error made when we use all nine formulæ and take the mean of their results shows that, as a rule, we shall obtain less error by selecting one good formula like (8) or (9) and using that only than if we attempt to use them all. In round numbers we see that the mean error made in reconstruction by these formulæ is about 5 per cent., but if we use (8) or (9) the mean error will lie between 3 and 4 per cent. The maximum error reached by a good formula like (8) or (9) is upwards of 10 per cent., but its occurrence is infrequent. On the whole, I consider this reconstruction of the individual from data for his own race fairly satisfactory. It is practically nearly as good as we get in the reconstruction of stature from the long bones.† I would also remind the reader that the theory of correlation shows that we cannot hope to get better results. We have solved the problem as closely as it can be solved, so long as the skull is a variable organ. From a knowledge of the degrees of variation and correlation of an extended number of parts of the skull (unpublished data), I feel fairly confident that no external measurements can be taken upon it which will give substantially better results than those already considered.‡ When we bear in mind that two different observers, using even

* If we wish to identify criminals, we select characters to be measured and indexed which exhibit the least correlation. In the same way to differentiate and specify races, it is best to select a group of characters having the least correlation; one such is certainly the cephalic index.

† See PEARSON: "On the Reconstruction of the Stature of Prehistoric Races" ('Phil. Trans.,' A, vol. 192, pp. 188–189).

‡ An appendix is devoted to a consideration of the horizontal and vertical girths.

TABLE XI.—Aino Male.

Table of Differences of Actual and of Reconstructed Skull Capacity.

No.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.		18.	19.	20.	Actual mean error.	Mean error calculated from probable error.
(1)	− 6	+ 79	+ 47	−26	+ 14	+ 85	−67	+40	+119	+45	+49	− 7	+120	+115	−87	+ 65	− 85	−46	+ 23	+45	58·50	36·16
(2)	− 85	+117	+167	−27	− 21	−170	+33	−12	+115	−22	+58	+ 78	− 36	− 11	+14	+ 26	−123	−96	+ 50	−39	65·00	66·47
(3)	− 65	+110	+218	−17	+130	+138	− 5	−62	+158	−47	+55	+103	− 82	+ 30	+ 3	+ 68	−115	+ 8	+ 33	−35	74·10	67·34
(4)	+ 18	+ 60	+242	−20	+ 12	+192	− 5	+11	+171	+11	+43	+ 47	+ 44	+ 64	− 2	− 26	−138	−40	+112	−66	66·20	76·40
(5)	− 31	+ 95	+ 31	−26	+ 6	+ 79	−57	+34	+106	+37	+53	+ 2	+ 95	+ 93	−81	+ 76	− 82	−60	+ 9	+52	55·25	32·63
(6)	− 17	+ 87	+ 51	−26	+ 45	+ 74	−69	+22	+121	+33	+51	+ 5	+ 93	+110	−85	+ 81	− 82	−30	+ 11	+49	58·60	35·03
(7)	−100	+131	+153	−19	+ 81	+123	+ 4	−50	+123	−42	+51	+ 94	− 86	+ 3	− 3	+ 83	−105	−36	+ 7	− 9	65·65	59·32
(8)	− 37	+100	+ 34	−26	+ 28	+ 79	−60	+22	+108	+29	+54	+ 9	+ 76	+ 91	−81	+ 67	− 80	−67	+ 1	+54	55·15	31·96
(9)	− 78	+117	+105	−27	+ 82	+ 81	−34	−34	+116	−22	+55	+ 58	− 35	+ 41	−43	+100	− 92	−29	− 6	+25	59·00	50·74
Mean deviation	− 49	+100	+116	−24	+ 42	+ 76	−29	− 3	+126	+ 2	+53	+ 43	+ 21	+ 60	−41	+ 65	−100	−46	+ 60	+ 8	53·20	
Actual capacity	1485	1385	1190	1475	1465	1245	1425	1470	1300	1480	1415	1380	1465	1440	1430	1490	1610	1525	1350	1555		

TABLE XII.—Aino Female.

Table of Differences of Actual and of Reconstructed Skull Capacity.

No.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	Actual mean error.	Mean error calculated from probable error.
(1)	−133	−23	+ 8	+57	− 75	−19	+ 70	+63	− 20	+ 18	+ 38	+77	+ 41	+143	− 85	−35	+93	−55	− 4	+97	57·70	53·63
(2)	− 99	−63	+ 6	+91	− 56	−48	+119	− 4	−149	−104	+107	− 3	+ 18	+ 39	−116	−69	−11	−18	−47	+26	59·65	61·93
(3)	− 78	−32	+26	−42	−100	+ 9	+ 36	−47	−154	− 96	+113	+41	+ 47	+ 7	− 46	−49	+21	+64	+79	− 6	54·65	61·12
(4)	−139	+ 1	+ 5	+51	− 52	+16	+133	+18	−133	− 64	+131	+49	+132	+ 45	− 42	−58	+59	+18	+47	+27	61·00	69·41
(5)	−112	−46	+13	+78	− 71	−39	+ 69	+57	− 25	+ 3	+ 34	+59	+ 2	− 29	−110	−35	+66	−64	0	+77	49·45	49·95
(6)	− 94	−25	+29	− 1	− 97	− 2	+ 17	+32	− 25	+ 10	+ 39	+89	+ 19	− 48	− 66	−19	+87	−10	+23	+91	41·15	48·85
(7)	− 52	−61	+33	+ 5	− 93	−24	+ 32	−31	−126	− 88	+ 87	+28	− 16	− 4	− 89	−42	0	+28	+67	− 8	45·70	53·38
(8)	− 75	−45	+33	+22	− 93	−22	+ 18	+29	− 30	− 3	+ 36	+72	− 18	− 46	− 93	−20	+62	−20	+25	+72	41·70	54·91
(9)	− 76	−56	+33	+ 4	−102	−27	+ 6	+11	− 37	− 15	+ 35	+64	− 30	− 50	− 97	−22	+46	−17	+27	+59	40·70	44·84
Mean deviation	− 95	−39	+21	+29	− 82	−17	+ 56	+14	− 78	− 38	+ 69	+53	+ 22	+ 6	− 83	−39	+47	− 8	+24	+48	43·4	
Actual capacity	1450	1340	1320	1260	1370	1325	1170	1330	1500	1430	1170	1305	1210	1250	1380	1395	1300	1295	1255	1340		

TABLE XIII.—German Male.

Table of Differences of Actual and Reconstructed Skull Capacity.

No.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	Actual mean error.	Mean error deduced from probable error.
(1)	−187	− 52	− 53	− 48	−82	− 94	+ 86	−67	+18	− 27	+63	+68	−11	+65	+122	+ 83	+172	+ 79	+ 5	+159	77·05	79·94
(2)	−181	− 36	− 62	+ 7	−62	−101	− 49	+ 3	−97	− 82	+36	−30	+88	−16	+ 32	+ 39	−146	+164	+ 78	+ 63	68·60	69·07
(3)	−179	−160	− 93	− 62	−90	− 20	− 18	−30	+26	+ 15	+14	+29	+60	+72	+ 54	+105	+119	+177	+196	+187	85·30	90·48
(4)	−197	−181	−142	−109	−91	− 70	− 96	−10	−28	+ 5	+21	+16	+85	+48	+ 53	+ 87	+ 41	+171	+208	+219	93·90	91·35
(5)	−181	− 68	− 16	+ 38	−58	−114	+ 47	−27	−72	− 98	+58	− 2	+38	− 7	+ 69	+ 37	− 75	+117	− 25	+ 32	58·95	61·50
(6)	−170	− 52	+ 5	+ 28	−73	− 36	+140	−67	+46	− 34	+47	+51	− 7	+66	+ 94	+ 90	+164	+115	+ 16	+ 85	69·30	75·07
(7)	−163	−139	− 28	+ 52	−58	− 51	− 18	+ 3	−76	− 83	+24	−40	+92	+13	+ 13	+ 45	−148	+190	+ 94	+ 21	67·56	65·62
(8)	−160	− 67	+ 29	+ 96	−52	− 66	+ 93	−29	−45	−100	+45	−13	+49	− 3	+ 49	+ 45	+ 71	+145	+ 15	− 22	59·70	89·87
(9)	−149	+ 89	+ 37	+106	−59	− 21	+105	−33	− 6	− 68	+27	− 6	+32	+22	+ 36	+ 68	− 11	+169	+ 49	+ 15	55·40	65·55
Mean deviation	−174	− 74	− 36	+ 12	−69	− 64	+ 32	−29	−26	− 52	+37	+ 8	+47	+29	+ 58	+ 66	+ 21	+147	+ 71	+ 84	56·8	
Actual capacity	1705	1660	1640	1625	1600	1572	1560	1535	1500	1485	1475	1460	1450	1433	1425	1405	1375	1360	1325	1260		

TABLE XIV.—German Female.

Table of Differences of Actual and Reconstructed Skull Capacity.

No.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	Actual mean error.	Mean error deduced from probable error.
(1)	− 70	− 76	− 90	− 77	− 1	−34	−40	−20	+ 1	+ 4	+20	+102	−10	+60	+84	−31	+129	+ 68	+ 85	− 95	54·85	63·02
(2)	− 92	−107	− 61	+ 43	− 48	+23	−27	−42	−35	− 15	+13	− 91	+71	+71	+48	−15	− 16	+ 39	+103	+141	55·05	61·37
(3)	−207	− 89	− 43	−186	− 58	−33	+ 1	+17	+ 2	+ 22	−20	+ 79	+44	+65	+80	+72	+ 76	+111	+162	+136	75·15	77·43
(4)	−172	−152	−108	−100	−117	−55	−45	−24	−16	+ 7	+16	+ 45	+52	+71	+85	+96	+117	+136	+151	+224	89·45	86·72
(5)	− 45	− 71	− 64	+ 19	− 7	+19	−83	−34	−11	+ 56	+17	− 8	+23	− 4	+62	−64	+ 57	+ 25	+ 74	− 47	39·50	51·06
(6)	− 97	− 48	− 55	−122	+ 2	−16	−86	+ 4	+19	+103	− 1	+119	−10	+60	+85	−29	+101	+ 62	+ 41	−114	58·70	59·23
(7)	−115	− 73	− 27	− 47	− 30	+28	− 3	−15	−21	− 1	− 8	− 51	+62	−20	+51	−17	+ 2	+ 35	+ 53	+ 90	37·45	56·85
(8)	− 67	− 50	− 39	− 28	− 5	+23	−60	−15	− 3	+ 59	+ 2	+ 12	+21	0	+63	−60	+ 40	+ 23	+ 42	− 64	36·05	48·22
(9)	−100	− 41	− 20	− 98	− 5	+17	−42	+ 5	+ 5	+ 63	−15	+ 42	+16	+15	+66	−40	+ 34	+ 35	+ 19	− 48	36·30	50·38
Mean deviation	−107	− 79	− 56	− 66	− 30	− 3	−43	−14	− 6	+ 33	+ 3	+ 28	+30	+35	+69	−10	+ 60	+ 59	+ 81	+ 25	41·8	
Actual capacity	1520	1490	1444	1433	1415	1390	1378	1362	1355	1335	1322	1300	1280	1270	1255	1240	1225	1202	1185	1100		

the same process—if they have not worked together, watching and comparing each other's methods—may easily differ by 20 to 40 cubic centims. in their determination of skull capacity for the same skull, we appreciate that the errors made by our reconstruction formulæ are not much greater than the personal equation of two observers.

We may then conclude that our formulæ will allow us to make from the usual measurements of L, B, and H a fair estimate of the capacity of a skull, which is too fragile or too imperfect to have its capacity determined directly.

(9.) The next problem under this section is: *The determination of the individual capacity from data drawn from a second race.* This really involves the second fundamental problem, but for purposes of practical convenience I consider it here, justifying my application later. I found very poor results arose when I calculated individual Germans from Aino formulæ except in the case of formula (9). This on the other hand gave almost as good results, as if the individual Germans had been determined directly from their own racial formula (9). To illustrate this, I give in Tables XV. and XVI. the reconstruction for the selection made at random of forty German skulls, and further, a reconstruction for forty Naqada skulls also taken at random. In both cases I calculated the capacity from the Aino formula (9). German formula (9) applied to the Naqada gave very nearly identical results.

TABLE XV.—German Capacity calculated from Aino Formula (9).

Male.			Female.		
Actual capacity.	Calculated.	Difference.	Actual capacity.	Calculated.	Difference.
1705	1558	−147	1520	1417	−103
1660	1573	− 87	1490	1458	− 32
1640	1678	+ 38	1444	1422	− 22
1625	1733	+108	1433	1329	−104
1600	1545	− 55	1415	1407	− 8
1572	1554	− 18	1390	1405	+ 15
1560	1667	+107	1378	1330	− 48
1535	1505	− 30	1362	1362	0
1500	1497	− 3	1355	1356	+ 1
1485	1421	− 64	1335	1395	+ 60
1475	1506	+ 31	1322	1300	− 22
1460	1457	− 3	1300	1337	+ 37
1450	1496	+ 46	1280	1289	+ 9
1433	1459	+ 26	1270	1276	+ 6
1425	1465	+ 40	1255	1314	+ 59
1405	1476	+ 71	1240	1188	− 52
1375	1369	− 6	1225	1250	+ 25
1360	1532	+172	1202	1226	+ 24
1325	1388	+ 63	1185	1192	+ 7
1260	1280	+ 20	1100	1034	− 66
	Mean error = 56·7			Mean error = 35·0	

TABLE XVI.—Naqada Capacity calculated from Aino Formula (9).

Male.			Female.		
Actual capacity.	Calculated.	Difference.	Actual capacity.	Calculated.	Difference.
1448	1418	− 30	1266	1271	+ 5
1354	1375	+ 21	1174	1171	− 3
1354	1379	+ 25	1148	1146	− 2
1260	1351	+ 91	1195	1213	+ 18
1481	1502	+ 21	1160	1228	+ 68
1232	1285	+ 53	1120	1223	+103
1335	1329	− 6	1248	1209	− 39
1388	1430	+ 42	1451	1383	− 68
1326	1288	− 38	1160	1268	+108
1338	1348	+ 10	1290	1276	− 14
1305	1413	+108	1106	1124	+ 18
1224	1366	+142	1214	1159	− 55
1368	1380	+ 12	1120	1249	+129
1328	1321	− 7	1190	1280	+ 90
1475	1435	+ 40	1304	1276	− 28
1281	1305	+ 24	1173	1215	+ 42
1440	1426	− 14	1152	1137	− 15
1174	1252	+ 78	1135	1173	+ 38
1292	1321	+ 29	1299	1285	− 14
1253	1374	+121	1158	1152	− 6
	Mean error = 45·6			Mean error = 43·15	

Now Tables XIII. and XIV. show that the mean errors made for the 20 ♂ and 20 ♀ German skulls, reconstructed by the German formulæ (9) were respectively 55·4 and 36·3 cub. centims. The same skulls reconstructed from the Aino formulæ (9) give mean errors of 56·7 and 35·0 cub. centims.; while the Naqada skulls have mean errors of 45·6 and 43·1 cub. centims. respectively. We may thus conclude that within the limits of error occurring in reconstructing capacity, formula (9) as found for any race may be safely used to calculate the capacity of an individual of a different race. This is a very important result, and its basis will be further considered in the next section of this paper. We conclude that an average error of, say, 3 to 4 per cent. is all we shall make in applying (9) to determine the skull capacity of any individual not necessarily of the same race.

(10.) *Second Fundamental Problem. On the determination of the mean skull capacity of any local race of man from the regression formulæ of a second race.*

Professor KARL PEARSON has shown in a memoir, not yet published, that a general theorem holds for the influence of selection on the regression formulæ. A statement of this theorem is given by him in the 'Phil. Trans.,' A, vol. 192, p. 177. It may be summed up as follows: If selection has differentiated local races, then the regression formulæ will in general change from local race to local race, but that certain *indirectly* selected organs, when they have their values expressed in terms of *all* the directly selected organs, and any number of indirectly selected organs will have regression formulæ the same for the differentiated races. Further, if size be the character chiefly selected, then the changes in the constants of the regression formulæ will only be of the second or third order.

Without entering into a discussion of this and allied theorems by which Professor PEARSON hopes to quantitatively attack the problem of the evolutionary relationship of local races, I would note that for our immediate purposes we seek a formula which will apply to all local races, and that the best formula will be one that is sensibly identical in its results for extremely different types of life.

Now a very short inspection of Tables V. to VIII. shows that for neither sex are the constants for any one of the first eight regression formulæ approximately alike. It seems therefore absolutely impossible to apply successfully any one of these to any other local race. On the other hand, considering the comparative paucity of the skulls dealt with, there is a remarkable agreement between the constants of formula (9) for both races. This agreement for different races again receives striking confirmation when we examine the results for the Naqada race given on p. 237. I reproduce the whole series here :—

TABLE XVII.—Reconstruction Formula (9).

Males.

German formula . . .	$C = \cdot000332 \times L \times B \times H + 415\cdot34$
Aino formula	$C = \cdot000328 \times L \times B \times H + 430\cdot30$
Naqada formula . . .	$C = \cdot000352 \times L \times B \times H + 372\cdot39$
Mean formula	$C = \cdot000337 \times L \times B \times H + 406\cdot01$

Females.

German formula . . .	$C = \cdot000383 \times L \times B \times H + 242\cdot19$
Aino formula	$C = \cdot000400 \times L \times B \times H + 187\cdot80$
Naqada formula . . .	$C = \cdot000416 \times L \times B \times H + 189\cdot81$
Mean formula	$C = \cdot000400 \times L \times B \times H + 206\cdot60$

We could hardly have selected three more diverse races than German, Aino, and Naqada, and yet we have reached for sparse material a surprising identity of results! If we want the mean skull capacity of any race for which L, B, and H are known, we have only to select the closest race out of Table XVII., or, failing knowledge of racial relationships, the mean formula, and we shall obtain a result well within the error of the personal equation of two observers, or the differences arising from using different methods of directly determining capacity. These points we shall now demonstrate numerically.

In Table XVIII. a comparative illustration is given of the accuracy of formula (9), and the failure of formulæ (5) to (8) when they are applied from one local race to a second; formula (9) alone comes out and comes out triumphantly from the test. The errors made are from 2 to 5 cubic centims. on a total of 1300 to 1500, or the largest error is less than ·45 per cent.

TABLE XVIII.

Formula.	Mean capacity of Germans calculated from Ainos.	Actual value.	Mean capacity of Ainos calculated from Germans.	Actual value.
	Male		Male	
(5)	1442·07		1433·59	
(6)	1392·44		1549·29	
(7)	1575·56		1376·13	
(8)	1445·00		1432·61	
(9)	1506·91	1503·72	1459·14	1461·64
	Female		Female	
(5)	1327·82		1285·92	
(6)	1268·97		1380·24	
(7)	1374·73		1240·01	
(8)	1324·77		1292·20	
(9)	1331·89	1337·15	1313·45	1307·69

TABLE XIX.—Reconstruction of Local Races from Formulæ (9).

Race.	Sex.	German formula.		Aino formula.		Naqada formula.		Mean formula.		Actual value.
		Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	
German	♂	—	—	1507	+ 3	1528	+24	1512	+ 8	1504
Aino	♂	1459	− 3	—	—	1479	+17	1466	+ 4	1462
Naqada	♂	1372	− 15	1375	− 12	—	—	1377	− 10	1387
Theban Mummies . .	♂	1368	− 22	1365	− 25	1379	− 11	1370	− 20	1390
Modern Egyptians . .	♂	1346	− 9	1349	− 6	1359	+ 4	1350	− 5	1355
Ancient Etruscans . .	♂	1427	− 29	1430	− 26	1445	− 11	1433	− 23	1456
German	♀	—	—	1332	− 5	1380	+43	1351	+14	1337
Aino	♀	1313	+ 5	—	—	1353	+45	1325	+17	1308
Naqada	♀	1246	− 33	1236	− 43	—	—	1255	− 25	1279
Theban Mummies . .	♀	1235	− 19	1224	− 30	1267	+13	1243	− 11	1254
Modern Egyptians . .	♀	1171	− 25	1158	− 38	1199	+ 3	1177	− 19	1196
Ancient Etruscans . .	♀	1294	− 30	1287	− 37	1332	+ 8	1305	− 19	1324

In Table XIX. a more elaborate investigation is made of formula (9) only, using the four forms given in Table XVII. and tabulating the errors made. We see that the maximum error of the mean formula is under 2 per cent., and the average error under 1 per cent. These errors appear to me less than the personal equation of two observers, measuring the same series of skulls. In fact, I am inclined to think that the errors of the mean formulæ may be as much due to the different observers who have determined the "actual" values as to defects in the formulæ themselves. The close association of the Aino and German results is specially noteworthy.

The results for the correlation and regression, not only of the skull, but of the long bones of the Ainos, show a relation much closer to modern Europeans (French and German) than the latter bear to the Naqadas. The primitive Aino race appears to be in some manner much more closely related to the evolutionary source of the Aryan races than either are to the Naqada.

On the other hand, it will be seen that the Naqada formula while giving bad results for German and Aino gives much better results than they do for both the ancient and modern Egyptians. Its maximum error as applied to the Egyptian races is only slightly over 1 per cent., while its average error as applied to all three Egyptian races is under ·4 per cent.

The mean formula over-estimates the Aino and German, and under-estimates the Egyptian races.*

The general rule for deducing the best result, would clearly be to work with the formula for the most closely associated race. But if no association can be predicted, then we shall hardly have an error as large as 2 per cent. if we use the mean formula. As this error is less than that frequently obtained by different observers for the same series, I conclude that a fairly satisfactory formula has been reached for the reconstruction of skull capacity from external measurements.

(11.) At this point it seems desirable to say a few words about the errors made by different observers in estimating skull capacity. I believe the differences of the same observer using different methods on the same skull can be reduced to a very few cubic centimetres, but the personal equation of two observers using different or even nominally the same methods on the same skulls will be very considerable. When the observers have been trained in different schools and use different methods the divergences may be very great. The value of the capacity depends so largely on the amount of "packing" both in the skull and in the measuring glass. Thus I found with two very careful investigators measuring about fifty skulls, their averages differed by about 30 cubic centims., and this difference was approximately constant for each skull. Three measurers using the same process with great care got results for individual skulls occasionally differing by even as much as 40 cubic centims.! On the

* It should be noticed that the German formulæ give better results than the Aino for the Naqadas, although in cephalic index the Aino is much closer to the Naqada than the German is.

other hand one measurer using the same method soon obtained practically identical results in making re-measurements, and even one measuring in three different ways.

If the reader will merely look at the Table XX. which follows, giving the capacity and chief dimensions of the skull for a number of races, he will easily convince himself that the differences in capacity must be largely due to the differences of personal equation and of method and not to the thicknesses of bone in the crania. Take the French (P) skulls; they are not decisively the largest in the series and yet they are credited with capacities which easily head the list. For relative purposes it is almost impossible to credit different series with a correctness within 30 cubic centims. of the true value. Hence such deviations as we find in the second column of Table XIX. seem well within the observational accuracy attainable, and I think it quite possible that if we had some further large series of L, B, H, and C, determined by careful observers, we should have a formula giving more trustworthy results for the mean capacity than could be obtained by the direct measurement of an individual observer. The averaging of a number of series would tend to eliminate the large personal equations which I feel sure exist in measurements of this kind.

TABLE XX.—General Table of Skull-dimensions for divers Races.

Race.	Number.	Sex.	L.	B.	H.	H′.	C.
Aino	76	♂	185·82	141·23	119·32	139·50	1462
Malay†	76	♂	174·33	142·36	116·88	140·68	1430
Negro*	54	♂	185·04	135·20	[115·17?]§	134·77	1430
Bavarian	100	♂	180·58	150·47	120·75	133·78	1503
Badenser†	78	♂	181·50	148·60	113·40	132·50	1525
French (M)‡	56	♂	179·96	143·41	112·86	128·95	1473
French (P)*	77	♂	182·69	145·22	[117·71?]§	132·01	1560
Egyptians, ancient†	201	♂	181·83	137·14	114·28	135·94	1390
Egyptians, modern†	76	♂	179·11	136·51	115·42	137·50	1355
Naqada	69	♂	185·13	134·87	115·59	135·21	1387
Etruscans†	78	♂	182·88	143·53	115·90	139·20	1456
Aino	52	♀	177·17	136·79	114·97	135·10	1308
Negro*	23	♀	174·52	130·52	[106·51?]§	126·91	1256
Bavarian	100	♀	173·59	144·11	114·17	128·01	1337
Badenser†	45	♀	172·20	141·30	107·70	124·90	1339
French (P)*	41	♀	174·34	135·49	[112·10?]§	125·10	1338
Egyptians, ancient†	96	♀	175·92	134·16	110·25	130·64	1254
Egyptians, modern†	23	♀	175·04	131·00	107·65	130·81	1196
Naqada	98	♀	177·48	131·61	113·11	129·55	1279
Etruscans†	38	♀	177·47	138·81	111·34	133·71	1324

* Extracted for Professor PEARSON from BROCA'S manuscript registers at Paris, by the courtesy of M. MANOUVRIER.

† From the German Anthropological Catalogue.

‡ Skulls of French prisoners who died in Munich during the Franco-German war. (German Anthropological Catalogue.)

§ Rough estimate, as data were wanting.

While I believe strongly in picking out the formula for the most closely associated race, I give the value of the constants for the male and female formulæ as obtained not by correlation, but by the method of least squares from the results in Table XX.

For the males excluding the negroes, I find for the ten races :*

$$♂ \quad C = \cdot000365\, L \times B \times H + 359\cdot34 \quad . \quad . \quad . \quad . \quad . \quad (10).$$

For the females excluding the negroes, I find for the eight races :

$$♀ \quad C = \cdot000375\, L \times B \times H + 296\cdot40 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Table XXI. gives the capacities of the races as found from (10) and (11).

Such equations should, I think, only be used when we have no knowledge of the evolutionary history of the race, which would lead us to adopt one of the special equations of Table XVII.

(12.) In attempting to use the formulæ given in this section, the reader must bear in mind that they can only be applied when the maximum length, maximum breadth, and the *auricular* height are known. The latter measurement unfortunately is occasionally omitted in series of skull measurements. If the total height of the skull H′ be given, then the following formulæ can be used, which have been calculated by the method of least squares from all the results in Table XX.

$$♂ \quad C = \cdot000266\, L \times B \times H' + 524\cdot6 \quad . \quad . \quad . \quad . \quad . \quad (12),$$

$$♀ \quad C = \cdot000156\, L \times B \times H' + 812\cdot0 \quad . \quad . \quad . \quad . \quad . \quad (13).$$

The following table includes results from (12) and (13) as well as from (10) and (11) :—

* Excluding for various reasons the unsatisfactory French (P), the French (M), and the Badenser, I find

$$C = \cdot000370\, L \times B \times H + 321\cdot16 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)\ bis.$$

TABLE XXI.—Calculated and Observed Capacity for 10 Races.

Race.	Sex.	From L, B, and H.	From L, B, and H′.	Observed.
Aino	♂	+40 (+17)	+ 36	1462
Malay	♂	−12 (−35)	+ 23	1430
Negro	♂	[−19?]	− 9	1430
Bavarian	♂	+54 (+32)	− 11	1503
Badenser	♂	−50	− 50	1525
French (M) . . .	♂	−51	− 63	1473
French (P). . . .	♂	−61	−104	1560
Ancient Egyptian .	♂	+10 (−15)	+ 36	1390
Modern Egyptian .	♂	+34 (+10)	+ 64	1355
Naqada	♂	+23 (+2)	+ 36	1387
Etruscan	♂	+14 (−9)	+ 40	1456
Mean deviation .	—	34·8 (17·1)	42·9	—
Aino.	♀	+34	+15	1308
Negro	♀	[−50?]	+ 7	1256
Bavarian	♀	+31	−25	1337
Badenser	♀	−60	−53	1339
French (P). . . .	♀	−49	−65	1338
Ancient Egyptian .	♀	+18	+39	1254
Modern Egyptian .	♀	+26	+82	1196
Naqada	♀	− 1	+ 5	1279
Etruscan	♀	+ 1	− 6	1324
Mean deviation .	—	24·6	33·0	—

The table illustrates several important points, thus:

(i.) We obtain less average error by estimating with H than H′, or the capacity of the skull is better determined from the auricular height, than from the total height of the skull.

(ii.) If we exclude the series for which the values of the capacity seem to be doubtful, *i.e.*, the Badenser and French, we obtain (bracketed numbers from (10) *bis*, footnote, p. 247) a mean error of about 1·2 per cent. and a maximum error of 2·5 per cent. For the series as a whole we have a mean error of about 2·4 per cent. with a maximum error of 4 per cent.

The latter occurs in the case of the Parisian French; but I have not the least hesitation in asserting that the capacities of the French skulls as determined in France. are quite incomparable with the capacities as determined by German investigators. I believe the French capacity is 60 to 80 cubic centims. beyond its true value, and I hold that my formula determines that value far more closely than the mean of the numbers (1560) given by BROCA's MS. registers. I do not think it can differ by more than a few cubic centimetres from 1499, and this difference is probably in defect. It will be seen that the Munich French skulls are somewhat smaller than the Parisian French skulls, but this does not account for the whole difference of 87 cubic centims. found by German and French determinations. It is largely a question of method. Again Mr. HERBERT THOMPSON found for the capacities of 39 ♂ and 55 ♀ Naqada skulls, 1339 and 1243 cubic centims. respectively, but Miss C. D. FAWCETT using a different method on 69 ♂ and 98 ♀ skulls obtained 1387 and 1279 cub. centims. respectively. Something here is due to the difference of the samples, but as in the previous case the personal equation is the chief source of difference. Now if differences of sample, of observer and of method will lead to determinations of racial capacity differing by 3 to 6 per cent., is not a great deal to be said for a formula which when applied to a series of results of a uniform character (like those of the best German determinations given above) leads to an error of only 2·5 per cent. as a maximum? I should personally feel as content with the results in Table XXI. of my mean regression formulæ and of the least square formulæ of p. 247, as with the average found for a race after days of laborious determination of capacity by aid of shot, seed, or sand. If the reader be not content with this degree of approximation, then I think no formula will satisfy him; *for nature being inherently variable, the capacity is no definite function of any dimensions of the skull, it is only moderately correlated with these dimensions, and the probable error of the determination cannot be reduced beyond quite sensible limits.*

The alternative to a formula is, of course, to make direct determination more uniform and exact. Now I believe two observers may be trained to get fairly accordant results, but will these results be the *real* capacity of the skull? May not the reality lie more nearly in the mean of the determinations of a number of careful observers measuring independently? Their errors may fall on either side of the truth, whereas a systematised procedure may give their errors a common bias. Hence a formula based upon a fairly wide set of results by different, but careful, observers may after all be more trustworthy than direct determination by a conventional method. It might, of course, be possible to reduce the conventional method to physical exactness; but I do not think this exactness is reached by the construction of control skulls (*Normalschädel*, *Crâne étalon*), which cannot cover all types; it might possibly be done by opening each skull (allowing for the thickness of the saw cut), and then filling either half. But such a process is laborious, it destroys the skull for some other purposes, and when the true capacity has been found we should have only the *average of a sample*. With the size of cranial samples at present available, the mean errors of the means amount to about 12 cubic centims., or are of the order of the errors of a good formula. Hence physical exactness (which would also improve the constants of the formula) is not all that is wanted.

(13.) Accepting the product formula as a working result, a further question may still arise as to whether it is needful to form the mean product of L × B × H or whether we may content ourselves with the product of the mean values of L, B, and H for the race.

The following table indicates the order of error made by using the product of means for the mean product :—

TABLE XXII.

Race.	Mean product.	Product of means.
Etruscan ♂	3,046,886	3,042,232
Etruscan ♀	2,746,817	2,742,818
German ♂	3,282,338	3,280,662
German ♀	2,860,213	2,856,635
Naqada ♂	2,881,137	2,886,107
Naqada ♀	2,619,631	2,642,039
Aino ♂	3,144,287	3,129,831
Aino ♀	2,797,032	2,786,983
Thebans ♂	2,859,374	2,849,705
Thebans ♀	2,589,815	2,602,057
Modern Egyptians ♂ . .	2,801,990	2,822,055
Modern Egyptians ♀ . .	2,424,920	2,468,440

It will be found that whether we use the mean product or the product of the means will make only a few cubic centimetres difference in the estimate, something under the 1 per cent., within which we cannot suppose our results to be correct. Hence for practical purposes we may content ourselves with using the product of the means, the determination of which is far less laborious. Our least square formulæ have all been based on the product of the means.

(14.) *Third Fundamental Problem. To reconstruct from external measurements an organ not measurable on the living organism,* i.e., *the skull capacity from measurements on the living head.*

It has been shown by KARL PEARSON ('Phil. Trans.,' A, vol. 192, p. 183) that if x and y be two characters and m, n, m', n' four constants, then the correlation coefficient of $mx + n$ and $m'y + n'$ is the same as that of x on y. The regression coefficient will be the same if $m = m'$. Now in the case of length, breadth, height, l, b, and h measured on the living head we have differences from their values as measured on the skull depending on the thickness of the living tissues covering the skull. These tissues of course vary from individual to individual, but as the thickness of the tissues themselves are of the second order of small quantities as compared with the length, breadth, and height of the skull, we may safely assume that their variations will be of the like order compared to those of l, b, and h. We shall thus obtain a very fair approximation to the regression coefficients connecting the skull capacity with head-length, breadth, and auricular height, by using those already found for the like quantities measured on the skull. Thus we should have a formula (9) of the form

$$C - C_0 = \alpha(l - l_0) + \beta(b - b_0) + \gamma(h - h_0) \quad . \quad . \quad . \quad . \quad . \quad (A)$$

where l_0, b_0, h_0 are the mean length, breadth, and auricular height of the living head, and C_0, α, β, γ constants to be determined from the measurement of skulls.

Further, formula (9) takes the form

$$C = \epsilon(l - \delta_1)(b - \delta_2)(c - \delta_3) + \eta \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

where ϵ and η are to be determined from skull measurements, and δ_1, δ_2, δ_3 give the mean differences between head and skull measurements. What values are to be given to these quantities?

As we have seen, the constants ϵ and η of (B) do not vary very much from local race to local race, while, on the other hand, α, β, γ of (A) differ very considerably from race to race. We shall hardly expect, therefore, to obtain as good results from (A) as from (B). Let us accordingly take (B) first, and consider δ_1, δ_2, δ_3.

H. WELCKER* gives the following measurements for an average of thirteen males in middle life:—

Thickness of flesh at back of head = 6·8 millims.
" " middle of forehead = 4·3 millims.
" " top of crown = 5·9 millims.

The values at the side of the head and on the auricular orifices are not given. But the results seem to show an average of 11 to 12 millims. to be subtracted from the head measurements when we wish to get those of the skull.

MERKEL† gives 6 millims. as an average thickness of the tissues covering the skull. Thus WELCKER and MERKEL are in good agreement.

We can consider this matter from another standpoint. I can find no head measurements from Bavarians or Badenser to compare with my skull measurements in Table XX., but the following table gives some results from English sources:—

TABLE XXIII.—Mean Head Measurements.

Organ.	Male.			Female.	
	B.A.	Anatomists.	U.C. Staff.	B.A.	B.C. Students.
l_0	198·1	198·4	196·38	185·6	189·71
b_0	155·0	157·2	153·48	147·3	146·78
h_0	130·9	133·1	134·78	128·4	132·73
$\frac{1}{3}(l_0+b_0+h_0)$	161·3	162·9	161·55	153·8	156·41

The British Association measurements are averages obtained by myself from the values given for several years in the "Reports of the Anthropometric Committee" which are published in the 'Transactions.'‡ They cover quite a long series. The "anatomists" are the head measurements of thirty-five of the anatomists attending

* 'Schillers Schädel und Todtenmaske,' Braunschweig, 1883.

† 'Handbuch der topographischen Anatomie,' Bd. I., p. 12.

‡ Reports of Committee, 1889 . . . 1893.

the meeting of the Anatomical Society in Dublin, June 10, 1898. They were measured in the Anthropometrical Laboratory of Trinity College, and the data were published in the 'Journal of Anatomy' in 1898. The University College staff consist of twenty-five members only of the staff of University College, London, measured by Professor KARL PEARSON. The Bedford College students were measured by Miss C. D. FAWCETT, B.Sc., and myself, and were thirty in number. In all these cases there were undoubtedly a good many heads not of English origin, but this was especially the case at the Anatomical Congress, where a number of foreign *savants* were present. I should consider the British Association returns the most homogeneous and reliable for working with, but it is noteworthy to what an extent the Bedford College women exceed in size of head the women attending the British Association meetings.

Now it would be impossible to compare the l_0, b_0, h_0 of the British Association measurements directly with the L_0, B_0, H_0 of the Bavarians, for the latter belong to a far more brachycephalic race. But if we compare $\frac{1}{3}(l_0 + b_0 + h_0)$ with $\frac{1}{3}(L_0 + B_0 + H_0)$ we find a difference of 10·7 for ♂ and 9·8 for ♀. If we compare the corresponding results for the Aino, a race with somewhat the same degree of dolichocephaly, we find differences of 12·5 and 10·8 respectively. Although we cannot lay much stress on this reasoning which supposes $\frac{1}{3}(L_0 + B_0 + H_0)$ approximately constant for local races, still it confirms WELCKER and MERKEL'S results so far as it goes. I think, without differentiating between the sexes, we shall obtain reasonable results by considering that we must subtract about 11 millims. from the head measurements in order to obtain the corresponding skull measurements. This being so, we have the following fundamental equations deduced from the mean equation of p. 243, to find the capacity from measurements on the living head :—

$$\left.\begin{array}{ll} ♂ & C = \cdot 000337\,(l - 11)\,(b - 11)\,(h - 11) + 406\cdot 01 \\ ♀ & C = \cdot 000400\,(l - 11)\,(b - 11)\,(h - 11) + 206\cdot 60 \end{array}\right\} \quad . \quad . \quad . \quad (14)$$

If we use the British Association mean values in (14), we find that for the mean skull capacity of the British—no doubt English in the bulk—the values

♂ 1495 cubic centims. ♀ 1323·5 cubic centims.

There appears at present to be no satisfactory determination of the skull capacity of English men and women, and these results are, I believe, as reliable as any estimates yet formed.* The ratio of ♂ to ♀ skull capacity would thus be 1·13, corresponding well with the ratio of brain weights, 1·12, as determined by REID and PEACOCK, but considerably higher than the ratio for brain weights given by CLENDINNING and SIMS.

A rough sort of control formula for comparison with (14) may be obtained by substituting the British Association values for C_0, l_0, b_0, and h_0 in the equation

$$C - C_0 = \epsilon\,(l \times b \times h - l_0 \times b_0 \times h_0).$$

* See PEARSON, "Variation in Man and Woman," 'The Chances of Death,' vol. I, p. 328.

In this way we find :

$$\left.\begin{array}{ll} ♂ & C = \cdot000{,}337\, l \times b \times h + 140\cdot13 \\ ♀ & C = \cdot000{,}400\, l \times b \times h - 80\cdot62 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (15).$$

This formula merely assumes that the factor multiplying the product of length, breadth, height remains the same, whether these quantities are measured on the head or the skull.

We now turn to the discovery of linear formulæ corresponding to (8) of pp. 234, 236. Here we are met by the very obvious difficulty that unlike formula (9) the constants of formula (8) change much from local race to local race. If we take the formula for the Germans as being nearest akin to the English, we are met by the obvious fact that the constants change widely when we pass from a brachycephalic to a dolicho-cephalic race; the English, indeed, have a cephalic index nearer to the Ainos than to the Germans. Accordingly, in default of more ample data for striking a mean formula, I have inserted in (A) of p. 250, the mean values of the German and Aino constants. We thus have :—

$$\begin{array}{ll} ♂ & C - C_0 = 10\cdot1025\,(l - l_0) + 8\cdot0345\,(b - b_0) + 3\cdot709\,(h - h_0), \\ ♀ & C - C_0 = 7\cdot222\,(l - l_0) + 8\cdot4605\,(b - b_0) + 6\cdot300\,(h - h_0). \end{array}$$

Inserting the British Association mean values for l_0, b_0, and h_0, as well as the mean capacities found from (14), we have :—

$$\left.\begin{array}{ll} ♂ & C = 10\cdot1025\, l + 8\cdot0345\, b + 3\cdot709\, h - 2237\cdot52 \\ ♀ & C = 7\cdot222\, l + 8\cdot4605\, b + 6\cdot300\, h - 2071\cdot22 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

Another linear formula may be obtained in an entirely different manner by taking the tangent plane at the mean to the surface in (14). Thus the skull measurement surface is :—

$$C = \epsilon LBH + \eta,$$

and the tangent plane is

$$C - C_0 = \epsilon L_0 B_0 H_0 \left\{ \frac{L - L_0}{L_0} + \frac{B - B_0}{B_0} + \frac{H - H_0}{H_0} \right\}.$$

Now introduce the British Association values, remembering that $L_0 = l_0 - 11$, $B_0 = b_0 - 11$, $H_0 = H_0 - 11$, and we find :—

$$\left.\begin{array}{ll} ♂ & C = 5\cdot8185\, l + 7\cdot5600\, b + 9\cdot0796\, h - 2017\cdot96 \\ ♀ & C = 6\cdot4006\, l + 8\cdot1992\, b + 9\cdot5192\, h - 2294\cdot46 \end{array}\right\} \quad . \quad . \quad . \quad . \quad (17).$$

Equation (17) will be found to give results excellently in accord with (14); it is the linear formula most comparable with (14), yet the coefficients differ very widely from those of (16), the height which is least influential in (16) being now the most influential factor. It would have been satisfactory to find (17) more closely in agreement with (16), but the universality of (14) on which (17) is based, is quite wanting in (16).

Lastly, we may place here the linear formula found by taking the value of the German coefficients of formula (8), (pp. 234 and 235), and using British Association mean values, we have :—

$$\left.\begin{array}{ll} \text{♂} & C = 7{\cdot}348\, l + 10{\cdot}898\, b + 5{\cdot}228\, h - 2334{\cdot}17 \\ \text{♀} & C = 7{\cdot}065\, l + 10{\cdot}126\, b + 4{\cdot}848\, h - 2101{\cdot}81 \end{array}\right\} \quad . \quad . \quad . \quad . \quad (18).$$

The following table illustrates the degree of closeness of these various formulæ as applied to 17 selected heads of very different sizes. We observe that while the formulæ give considerable differences in the absolute capacities, especially in the case of the macrocephalic heads, the relative order of the heads as determined by all the formulæ is the same with but two exceptions. In the first place (14), (15), (17) and (18) give a relative order entirely the same, except for the slight displacement of Professor Howes under (18). For the females (16) is also entirely in accord with (14), (15), (17) and (18). The second displacement is that of Professor Weldon's head under (16), which alters its place by two. I can only account for this by the fact that Professor Weldon has a high cephalic index (82·7), and therefore the German formula was likely to give a better result than one based on the average of the German and of a less brachycephalic race like the Aino.

Table XXIV.—Skull Capacities from Living Head by Various Formulæ.

Name.	Formula.				
	14.	17.	15.	16.	18.
J. Lynn Thomas	1813	1789	1861	1785	1773
W. F. R. Weldon	1616	1616	1632	1533	1579
W. Ramsay	1581	1579	1594	1569	1572
A. Keith	1530	1530	1536	1557	1548
A. Platt	1501	1502	1501	1479	1481
G. B. Howes	1483	1485	1481	1458	1496
K. Pearson	1452	1454	1444	1398	1410
E. Barclay Smith	1408	1407	1396	1365	1396
J. Kollmann	1373	1370	1353	1332	1369
♀ Student 1	1647	1620	1697	1593	1587
♀ Student 4	1514	1507	1543	1471	1458
♀ Student 8	1488	1481	1512	1453	1440
♀ Student 12	1450	1447	1471	1442	1430
♀ Student 16	1368	1368	1376	1384	1388
♀ Student 20	1321	1321	1320	1318	1307
♀ Student 24	1302	1305	1299	1303	1284
♀ Student 28	1230	1227	1214	1225	1216

Thus of the two exceptions to complete accordance we see that only Professor WELDON'S head in the case of formula (16) presents any serious disturbance of the relative order.

On the whole, my methods will, I think, determine within reasonable limits the relative order of skull capacity from measurements on the living head. It is noteworthy that except for the macrocephalic heads of Mr. LYNN THOMAS and Bedford College student No. 1, formulæ (17) and (14) give sensibly identical results, or there is one linear formula which gives results sensibly identical with those of the product formula. This shows us that the surface represented by (14) is sensibly plane for the range of skull measurements actually occurring. On consideration accordingly we may conclude that (14) (or its linear form (17)) gives the best results; (15) gives a good control formula; while of formulæ directly obtained from the regression equation for length, breadth, and height, the German appears best for the males, the mean of the German and Aino best for the females. For the remainder of my investigations on the capacity of the living head I shall accordingly use only the formulæ (14) and (16) or (18) for comparison.

I propose first to investigate whether there is any obvious relationship between skull capacity and current appreciation of intellectual ability.

My first series is contained in Table XXV. We have here the estimated skull capacities of thirty-five living anatomists. The list contains the names of many of great scientific reputation, and of others of less distinction. It will be seen that about the middle of the list, if we divide at D. HEPBURN, the eighteenth man, certain transfers would occur from the upper to the lower half, and *vice versâ*, if we judged by formula (18) and not (14). But these transfers are of men having roundly about the same skull capacity, and I think that generally we may feel quite satisfied with the accordance of the two series.* Now the average capacity of the first eighteen anatomists is 1601 cub. centims., and of the last seventeen anatomists is 1468 cub. centims. There is thus a most substantial difference.† Yet he would be a bold man who would assert that there is a substantial average intellectual superiority in the first half. In fact, a number of most capable men fall into the last nine, and J. KOLLMANN, one of the ablest living anthropologists, has absolutely the smallest skull capacity!

My second list contains the estimated skull capacity of twenty-five members of the teaching staff of University College, London. I give here the actual head measurements, as possibly of service in the future; those of the anatomists are published in the 'Journal of Anatomy' (see above). Here the first thirteen have a mean skull capacity of 1579 cubic centims. and the last twelve of 1436 cubic centims.—again a

* We must always remember that (14) is *à priori* to be considered a much better formula than (18), for the change of its constants from race to race is far less.

† The mean of the whole series as given by (14) is 1537, and by (18) is 1534, a remarkable accordance in the average results of the two methods.

TABLE XXV.—Estimated Skull Capacity of 35 Anatomists.

Name.	Formula (14).	Formula (18).
J. Lynn Thomas	1813	1773
A. H. Young	1656	1640
B. A. Windle	1649	1605
D. G. Cunningham	1635	1600
Hector Leboucq	1631	1654
C. de Bruyne	1616	1636
T. Symington	1604	1627
A. M. Paterson	1595	1616
E. H. Taylor	1593	1624
Wilhelm His	1587	1556
C. R. Browne	1585	1578
G. Elliott Smith	1573	1570
C. D. Marshall	1570	1561
F. Frohse	1569	1625
A. F. Dixon	1541	1513
R. J. Berry	1539	1538
A. Robinson	1538	1532
D. Hepburn	1531	1537
Arthur Keith	1530	1548
Anonymous	1520	1524
Robert Howden	1511	1498
G. Disse	1507	1519
T. H. Bryce	1507	1491
Hans Gadow	1506	1483
Stanley Boyd	1499	1466
James Cantlie	1486	1496
G. B. Howes	1483	1496
Sir Wm. Turner	1469	1473
A. Macalister	1456	1458
W. Spalteholtz	1455	1524
G. D. Thane	1443	1413
James Musgrove	1425	1445
E. Barclay Smith	1408	1396
Peter Thompson	1385	1318
J. Kollmann	1372	1369

very sensible difference.* The only differentiation I feel able to make between the two divisions here is that six out of the second twelve are mathematicians, and no mathematician has here a head above the average. In the first group we find not the exact but the descriptive sciences and the arts. No generalisation can be drawn, however, from such narrow data. We have only the suggestion of a field for further enquiry.†

The agreement in Table XXVI. between the results of formulæ (14) and (18) is not so good as in the case of Table XXV., but the approximate general order is

* The mean of the whole table is 1511, which may be compared with the 1537 of the anatomists. Both are sensibly larger than the British Association mean.

† The data for 1000 Cambridge men classified according to head measurements, branch of study and academic distinction, are at present being investigated.

maintained, and only one interchange between the first and second groups would take place.

TABLE XXVI.—Head Measurements and Estimated Skull Capacity of certain Members of the Teaching Staff of University College.

Name.	Head Measurements.			Estimated Capacity.	
	L.	B.	H.	(14)	(18)
H. TONKS	201	154	145	1633	1579
F. W. GOODBODY	203	160	137	1621	1617
T. G. FOSTER	201	159	139	1619	1602
W. F. R. WELDON	193·5	160	143	1616	1579
M. TRAVERS	199	158	140	1607	1582
F. G. DONNAN	197	155	143	1597	1550
W. RAMSAY	202	157	136	1581	1572
A. W. PORTER	199	154	140	1575	1535
J. SULLY	202	156	135	1563	1556
H. R. KENWOOD	194	162	135	1561	1563
R. RUSSELL	202	155	134	1546	1540
W. A. OSBORNE	197	150	138	1513	1470
A. PLATT	197	153	134	1501	1481
E. H. STARLING	204	149	131	1483	1473
L. N. G. FILON	201	151	130	1473	1468
W. P. KER	190	154	134	1467	1441
E. C. C. BALY	201	144	135	1462	1418
K. PEARSON	191	150	135	1452	1410
M. J. M. HILL	193	152	132	1452	1430
G. E. PETAVEL	192	155	130	1451	1445
G. THANE*	195	150	130·5	1436	1415
H. T. HARRIS	188	154	131	1430	1410
G. H. FOWLER	187	153	128	1391	1376
SWALE VINCENT	193	153	123	1381	1394
G. U. YULE	187	144	131	1352	1294

My third and last series, that of Table XXVII., contains the estimated skull capacities of thirty women students of Bedford College. I arranged these students on a considerable personal experience of their work into three classes of ten each, representing clever, medium, and dull students. I then divided the skull capacity list into three sections—large, medium, and small capacity. I was totally unable to find any correspondence between these two divisions into three classes.

I have used in this case formulæ (14) and (16). They give results generally in very good agreement, the general order not being substantially modified when we pass from one series to the other. The mean found from (14) is 1390 cubic centims., and

* The values for L, B, H differ somewhat from those determined at the Dublin Anatomists' Congress, but they are, I believe, correct.

from (16) is 1376 cubic centims. These are in fairly good agreement. The average capacity is thus very sensibly larger than that of the British Association women (p. 251).

TABLE XXVII.—Head Measurements and Estimated Skull Capacity of 30 Bedford College Women Students.

Students.	L.	B.	H.	Formula (14).	Formula (16).
No. 1	200	157	141·5	1647	1593
„ 2	198	154	138	1565	1531
„ 3	196·5	149	140	1527	1491
„ 4	190	151·5	141	1514	1471
„ 5	187	151	143	1508	1458
„ 6	189	151	141·5	1507	1463
„ 7	195	144	142	1489	1450
„ 8	191	150	139	1488	1453
„ 9	200	145	135	1463	1450
„ 10	195	149	134	1456	1442
„ 11	194·5	144	139	1456	1427
„ 12	199	146	133·5	1450	1442
„ 13	190	150	135·5	1446	1424
„ 14	190	149	131	1393	1387
„ 15	192	155	124	1385	1408
„ 16	194	149	126	1368	1384
„ 17	187	148	130	1354	1350
„ 18	188	147	130	1352	1349
„ 19	180	152	129	1331	1327
„ 20	189	142·5	130	1321	1318
„ 21	186	147	128	1320	1322
„ 22	184	148	127	1306	1310
„ 23	187	145	127·5	1306	1309
„ 24	192	138	130	1302	1303
„ 25	187	137	133	1289	1276
„ 26	187	142	127	1276	1281
„ 27	187	138·5	127	1248	1251
„ 28	180	141	127·5	1230	1225
„ 29	186	135	127	1213	1214
„ 30	170	148	125	1200	1196

From my Tables XXV. to XXVII. I conclude that there is certainly no *marked* correlation between skull capacity and intellectual ability.

There is another standpoint, however, from which these things may be considered. I know of no measurements upon which a direct determination of the correlation of brain *weight* and skull capacity could be made. Of course, the two are not proportional; still, there can hardly be a doubt that they are highly correlated. Now, if two quantities are correlated with a third, it does not invariably follow that they will be correlated with each other.* Yet I take it that it is rather quantity than density of brain stuff which is at the basis of the current belief that size of brain is closely related to intellectual ability, and that any illustration of the absence of

* A child is correlated with both parents, but, unless there be sexual selection, the parents are not correlated with each other.

sensible correlation between skull capacity and intellectual ability will tend to weaken current conceptions as to a relationship between brain weight and intellectual ability. The whole problem of the relation of size of head to intellectual distinction as judged by popular standards is under investigation from wider data; meanwhile, I think we may conclude—

(i.) That there is no marked correlation between skull capacity and intellectual power in the case of either sex alone.

(ii.) That brain weight must have a very considerable correlation with skull capacity, and, therefore, our data present nothing to encourage the belief that there is a relation between brain weight and brain power.

(iii.) That arguments based on the relative brain weight of the two sexes as showing relative brain power require a more solid quantitative basis than they at present exhibit.*

(iv.) That such arguments as those of A. R. WALLACE against the evolution of man's intellectual powers by aid of natural selection turn wholly on the size of the brain. But it would not appear from the above results that skull capacity at any rate is a character closely correlated with intellectual ability in the individual, and, therefore it is quite conceivably not correlated with racial ability.

So soon as data are forthcoming connecting the skull capacity with brain weight, or still better, brain weight with head measurements, we shall be in a position to reconstruct brain weight from head measurements. I do not see that the error of the determination is likely to be much larger than that found in the case of skull capacity, but if it reached 8 to 9 instead, say, of 3 to 4 per cent., it would still be sufficiently approximate to give quite reasonable results for large numbers of individuals classified into big groups according to their ability. It is, I hold, only by such methods that we can hope to reach any quantitative certainty of a relation between brain power and brain size. Personally I am inclined to hold with Professor PEARSON that the complexity of the convolutions of the brain, and the variety and efficiency of its commissures, rather than its actual size, are the characters we might expect to differentiate race from race and sex from sex, and to have developed with man's civilisation.†

I am not unaware that a correlation has often been asserted between brain weight and ability on the ground of the actual measurement of the brain weights of a number of men of genius. But what is the average of such brains compared with? The average brain weight of the bodies which reach the dissecting rooms of our hospitals, a large proportion of which belong to the emaciated and worn out. Probably on the same basis a correlation between genius and body-weight could

* Before questioning whether man or woman (relatively to stature, body weight, or other character) has the greater brain weight, it seems desirable to settle whether brain weight in either sex alone, absolutely, or relatively to some other character, has anything to do with intellectual ability.

† 'Grammar of Science,' 2nd ed., p. 539.

easily be demonstrated. Or again because English women have a mean brain weight of 1235 grs. and French women of 1142 grs., are we to argue that English women are intellectually abler than French women? The fact is that to solve a problem of this kind we ought to keep within one fairly equally nourished class, and within one local race and actually correlate brain size and ability. I do not see how this can be done for brain weight, but it seems to me quite possible for the capacity of the brain chamber estimated from external measurements.

(15.) *Conclusions.* I have now completed the discussion of the three problems I proposed to investigate. It will be seen that the accuracy of predictions depends sensibly on two factors: (i) the existence of suitable data upon which the regression formulæ can be based and (ii) the number of measuremeuts used to form an estimate. Thus in the third fundamental problem we do not get as good absolute results as we might hope to do, because we have not really at present available the best data possible. Again in the first fundamental problem we cannot expect to reconstruct the capacity of the individual skull without a fairly large average error. For it is of the very essence of the principle of variation, on which evolution itself depends, that in any population we should have an array of skulls with the same length, breadth, and height, and yet having within certain limits a variety of capacities. All we can hope to say is, that with such a length, breadth and height such a capacity is most frequent. When we come to averaging a series, then we shall determine with far greater accuracy the mean of an array. Here the nature of the problem is, however, modified. The question is now how far can we apply results deduced from one local race to a second. We want in fact a "panracial" regression formula to replace our intraracial regression formula. As it is impossible to find such a regression formula for the primitive stock from which man may be supposed to be derived, we are compelled to take the regression formulæ which are least changed as we pass from race to race. The mean formula thence derived appears to give excellent results, when applied to determine the capacity of very diverse races. While I do not profess to have solved the problems proposed to the degree of accuracy which might be obtained with wider data and measurements made *ad hoc* in the anatomical school, I yet consider that I have given practical solutions to the following problems :—

(i.) The reconstruction of the capacity of the individual skull, when this cannot be measured directly. This is done with a mean error of 3 to 4 per cent.

(ii.) The determination of the mean skull capacity of a race without the use of sand, seed, or shot, to a degree of accuracy comparable with that of the direct method owing to the personal equation of the measurers even when using the same method of direct determination.

(iii.) The determination of the skull capacity of living individuals with a degree of accuracy sufficient to determine whether skull capacity is or is not closely correlated with intellectual power.

Appendix.

On the Correlation of Skull Capacity with Circumferential Measurements on the Skull.

It may have occurred to some readers that other measurements of the skull beside length, breadth, and auricular height would give effective means of reconstructing the capacity. The two that most readily suggest themselves are the horizontal circumference, U say, and the vertical circumference, from the top rim of one auricular passage over the top of the skull to the other, V say. The following are the values for C, U, and V for the Naqada skulls as measured and calculated by Miss C. D. FAWCETT, B.Sc., who has most kindly placed them at my disposal.

Naqada Skulls, ♂.

Organ.	Mean.	S. D.	Correlation.
U . . .	509·170	12·178	r_{UC} = ·6803
C . . .	1379·23	109·213	r_{UV} = ·5116
V . . .	304·423	9·850	r_{VC} = ·6736

Naqada Skulls, ♀.

U . . .	492·759	11·958	r_{UC} = ·6588
C . . .	1283·238	86·902	r_{UV} = ·4519
V . . .	296·615	8·430	r_{VC} = ·5821

The units are millims. for U and V and cubic centims. for C. From these the following equations for the reconstruction of C in terms of U and V result :—

For males :

$$C = 3{\cdot}5035\,U + 2{\cdot}7789\,V - 1250{\cdot}604$$

For females :

$$C = 3{\cdot}2244\,U + 3{\cdot}2859\,V - 1280{\cdot}286 \qquad (19).$$

I have worked out somewhat fuller data for the collection of skulls of *Theban Mummies* at Leipzig, the measurements of which are given in the German Anthropological Catalogue.

Ancient Egyptians, ♂, 202 Skulls.

Organ.	Mean.	S. D.	Correlation.
U . . .	511·722	14·010	$r = \cdot 8133 \pm \cdot 0161$
C . . .	1391·54	121·616	$r_{UV} = \cdot 6651 \pm \cdot 0265$
V . . .	306·703	8·204	$r_{VC} = \cdot 7876 \pm 0176$

Ancient Egyptians, ♀, 96 Skulls.

U . . .	495·104	14·116	$r_{UC} = \cdot 8262 \pm \cdot 0218$
C . . .	1251·98	102·063	$r_{UV} = \cdot 6246 \pm \cdot 0420$
V . . .	296·073	8·414	$r_{VC} = \cdot 6731 \pm \cdot 0377$

From these data I have deduced the following equations for reconstruction :—

For males :

$$C = 7{\cdot}060\ U - 2220{\cdot}98 \qquad \text{p.e.} = \frac{47{\cdot}72}{\sqrt{n}} \quad . \ . \ . \ . \quad (20)^a.$$

$$C = 11{\cdot}676\ V - 2189{\cdot}61 \qquad \text{p.e.} = \frac{50{\cdot}54}{\sqrt{n}} \quad . \ . \ . \ . \quad (21)^a.$$

$$C = 4{\cdot}505\ U + 6{\cdot}559\ V - 2925{\cdot}31 \qquad \text{p.e.} = \frac{39{\cdot}28}{\sqrt{n}} \quad . \ . \ . \ . \quad (22)^a.$$

For females :

$$C = 5{\cdot}974\ U - 1705{\cdot}73 \qquad \text{p.e.} = \frac{38{\cdot}78}{\sqrt{n}} \quad . \ . \ . \ . \quad (20)^b.$$

$$C = 8{\cdot}165\ V - 1165{\cdot}66 \qquad \text{p.e.} = \frac{50{\cdot}91}{\sqrt{n}} \quad . \ . \ . \ . \quad (21)^b.$$

$$C = 4{\cdot}811\ U + 3{\cdot}124\ V - 2054{\cdot}94 \qquad \text{p.e.} = \frac{36{\cdot}23}{\sqrt{n}} \quad . \ . \ . \ . \quad (22)^b.$$

Now, although the Naqada and Theban skulls have in some cases very close mean values—and it is impossible not to consider the races very closely related—yet the reconstruction equations for C from U and V differ very widely. It is true that the Theban skull capacity calculated from the Naqada formula or the Naqada capacity calculated from the Theban formula do not give very bad results :

	Actual.	From Theban formula (22).
Naqada { ♂ . .	1379	1365
Naqada { ♀ . .	1281	1242
	Actual.	From Naqada.
Theban { ♂ . .	1391	1394
Theban { ♀ . .	1251	1285

But this agreement does not arise from any real accordance in the formulæ, but from the fact of the close equality of the Naqada and Theban mean values for U, V and C.

To test the applicability of these circumferential formulæ when extended from one race to a second, I take the following data :—

Organ.	Race.		
	♂ Aino.	♀ Aino.	♂ French.*
U	522·5	501·7	527·6
V	328·5	317·1	317·9
C	1462	1308	1475

These lead to the following results for capacity :—

Race.	Actual.	From Naqada formula (19).	From Theban formula (22).
Aino ♂ . . .	1462	1493	1583
Aino ♀ . . .	1308	1379	1350
French ♂ . .	1475	1482	1537

We see :

(i.) That the Naqada and Theban formulæ, although deduced from kindred races and from very considerable numbers, lead to widely divergent results.

(ii.) That the Naqada, which is for Aino ♂ and French ♂ better than the Theban formula, gives results worse than the formulæ based upon L × B × H previously discussed.

We conclude, therefore, that it appears unlikely that a reconstruction formula, based on the circumferential measurements of the skull, can be found which will give good results, if extended from one local race to another.

If we apply these formulæ to reconstruct the capacity within the race, (20) and (21) give differences much of the order of the earlier reconstruction formulæ (1) to (8), while (22) gives results as good as (9).

The following table gives the errors made in estimating the capacity of forty Theban skulls, twenty of either sex, chosen at random. It will be seen that the errors can be fairly large when we use circumferential measurements.

* From the skulls of 50 French prisoners who died at Munich during the Franco-German war. Data given in the German Anthropological Catalogue.

Actual Values compared with Values Predicted from Circumferential Measurements in the case of 40 Theban Skulls.

No.	Male skulls.				Female skulls.			
	Actual value.	Error by (20)[a].	Error by (21)[a].	Error by (22)[a].	Actual value.	Error by (20)[b].	Error by (21)[b].	Error by (22)[b].
1	1480	− 2	− 50	− 10	1280	+ 1	+ 37	+ 20
2	1383	+ 32	− 23	+ 5	1337	− 74	− 69	− 70
3	1563	− 42	− 203	− 107	1356	− 15	− 48	− 11
4	1380	+ 70	− 7	+ 39	1253	+ 22	− 10	+ 14
5	1543	− 79	− 8	− 25	1413	− 90	− 23	− 51
6	1390	− 32	+ 28	− 4	1420	− 109	− 103	− 96
7	1310	+ 21	+ 74	+ 38	1227	+ 24	+ 65	+ 39
8	1355	− 61	− 18	+ 7	1120	+ 23	+ 66	+ 19
9	1353	− 44	− 60	− 98	1220	+ 121	+ 80	+ 122
10	1407	+ 29	− 35	− 2	1270	− 54	+ 14	− 35
11	1250	+ 143	− 77	+ 20	1333	− 142	− 8	− 102
12	1550	− 86	− 143	− 104	1330	− 133	− 62	− 116
13	1430	− 12	0	− 6	1260	+ 93	− 17	+ 70
14	1435	+ 22	− 17	+ 14	1390	+ 22	− 147	− 12
15	1493	− 50	− 40	− 34	1165	− 3	+ 94	+ 17
16	1250	+ 31	+ 75	+ 33	1195	− 3	+ 24	− 5
17	1290	+ 69	+ 23	+ 36	1093	− 31	+ 60	+ 24
18	1443	− 56	− 48	− 53	1347	− 6	− 22	+ 4
19	1170	+ 19	− 67	− 70	1245	+ 24	+ 31	+ 30
20	1525	+ 10	− 130	− 40	1250	− 76	+ 83	− 29
Mean error	—	45·5	56·3	37·25	—	53·3	53·15	44·3

Finally for our third problem—that of reconstructing the capacity of the living head—there appears no obvious method of allowing for the difference between the circumferential measurements with and without the living tissues. Of course such measurements as those now being made at Strasburg in the Anatomical Institute may surmount this difficulty and enable us to predict capacity from measurements on the living head.

It would thus seem that, as far as the present investigations go, circumferential measurements do not present great advantages over those discussed in the body of this paper, although the correlations between capacity and these measurements appear, as far as yet has been investigated, to have high values.*

* This is directly opposed to the view of Dr. Franz Boas ('American Anthropologist,' N.S., vol. I., p. 461). He holds that: "It would seem that circumferences are the most available means of judging cranial size." He does not appear, however, to have correlated the circumferential measurements with capacity, and seen how widely the resulting equations differ from race to race.