Bayes' Theorem: the relationship between the probability form and the odds ratio form

Bayes' theorem can be written in two different ways, in terms of probabilities, or in terms of odds ratios.

In the legal context we can use G to stand for guilty and E to stand for the evidence. What we want is the probability that the suspect is guilty in the light of the evidence. This is

$$\text{Prob}(\text{G}|\text{E}) = \frac{\text{Prob}(\text{G})}{\text{Prob}(\text{E})} \text{Prob}(\text{E}|\text{G})$$

This is a consequence of the rules of probability and it's exactly analogous to the form that's used for screening tests (see http://rsos.royalsocietypublishing.org/content/1/3/140216#app-1). Just substitute "test positive" for E (the evidence from the test) and "ill" for G (the presence or absence of illness is what you want to ascertain from the screening evidence).

We can derive the odds ratio form from this. First divide both sides by Prob(not G | E). The left hand side is now the posterior odds ratio in favour of guilt, *i.e.* odds in favour of guilt in the light of the evidence, relative to the odds in favour of innocence.

$$\frac{\text{Prob}(\text{G}|\text{E})}{\text{Prob}(\text{not G}|\text{E})} = \frac{\text{Prob}(\text{G})}{\text{Prob}(\text{E})} \frac{\text{Prob}(\text{E}|\text{G})}{\text{Prob}(\text{not G}|\text{E})}$$

The rules of probability give the bottom line on the right hand side as

$$\text{Prob}(\text{not G}|\text{E})\ \text{Prob}(\text{E}) = \text{Prob}(\text{E}|\text{not G})\ \text{Prob}(\text{not G})$$

Substitution of this gives

$$\frac{\text{Prob}(\text{G}|\text{E})}{\text{Prob}(\text{not G}|\text{E})} = \frac{\text{Prob}(\text{G})}{\text{Prob}(\text{not G})} \frac{\text{Prob}(\text{E}|\text{G})}{\text{Prob}(\text{E}|\text{not G})}$$